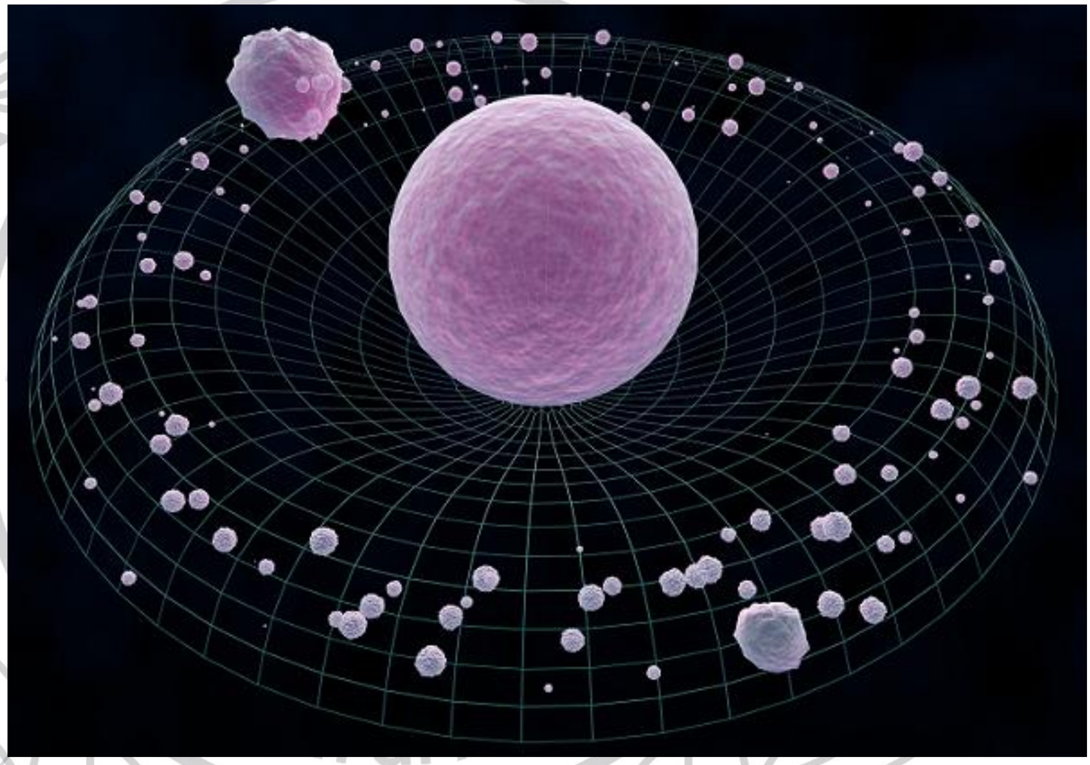
GRAVITATION

PART 1



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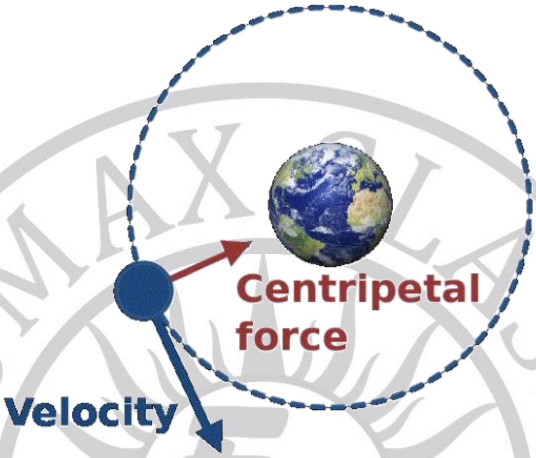
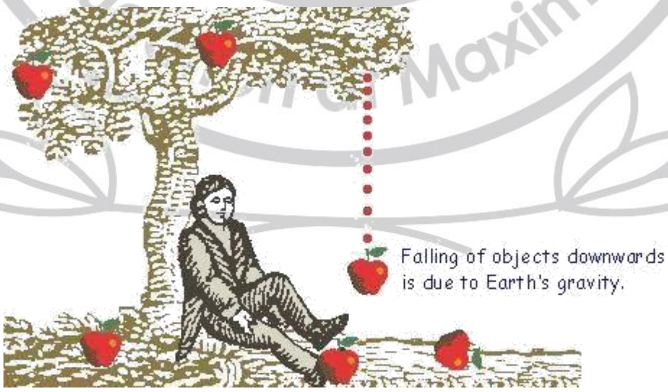
What is the Centripetal Force?

- We know that an object in circular motion keeps on changing its direction.
- Due to this, the velocity of the object also changes.
- A force called **Centripetal Force** acts upon the object that keeps it moving in a circular path.
- The centripetal force is exerted from the centre of the path.
- Without the Centripetal Force objects cannot move in circular paths, they would always travel straight.
- **For Example**, The rotation of Moon around the Earth is possible because of the centripetal force exerted by Earth.



Newton's Observations

- Why does Apple fall on Earth from a tree? – Because the earth attracts it towards itself.
- Can Apple attract the earth? - Yes. It also attracts the earth as per Newton's third law (every action has an equal and opposite reaction). But the mass of the earth is much larger than Apple's mass thus the force applied by Apple appears negligible and Earth never moves towards it.
- Newton thus suggested that all objects in this universe attract each other. This force of attraction is called **Gravitational Force**.



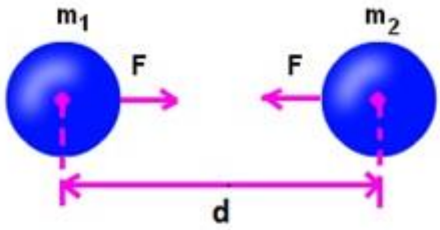
Gravitation

Earth attracts everything towards it by an unseen force of attraction. This force of attraction is known as gravitation or gravitational pull.

Universal Law of Gravitation

Every object in the universe attracts other object by a force of attraction, called gravitation, which is directly proportional to the product of masses of the objects and inversely proportional to the square of distance between them. This is called Law of Gravitation or Universal Law of Gravitation.

- This force is directly proportional to the product of their masses.
- This force is inversely proportional to the square of distances between them.
- Consider the figure given below. It depicts the force of attraction between two objects with masses m_1 and m_2 respectively that are 'd' distance apart.



- The figure below describes how the universal law of gravitation is derived mathematically.

$$F \propto m_1 \text{ ------ (i)}$$

$$F \propto m_2 \text{ ------ (ii)}$$

$$F \propto \frac{1}{r^2} \text{ ------ (iii)}$$

From the above equation we can rewrite them as the following:

$$F \propto \frac{m_1 m_2}{r^2} \text{ ------ (iv)}$$

If we remove the proportionality we get proportionality constant G as the following:

$$F = G\frac{m_1 m_2}{r^2}$$

The above equation is the mathematical representation of Newton's universal Law of gravitation

Hence, $G = Fr^2/ m_1 m_2$
SI Unit: $Nm^2\ kg^{-2}$
Value of $G = 6.673 \times 10^{-11}\ Nm^2\ kg^{-2}$ (was found out by Henry Cavendish (1731- 1810))

- The proportionality constant G is also known as the **Universal Gravitational Constant**

Why we study the universal law of gravitation?

It explains many important phenomena of the universe –

- Earth's gravitational force
- Why the moon always moves in a circular motion around the earth and the sun
- Why all planets revolve around the sun
- How the sun and moon can cause tides

Mathematical Expression for *g*
From the second law of motion, force is the product of mass and acceleration.

$F = ma$

For free fall, acceleration is replaced by acceleration due to gravity.

Therefore, force becomes:

$F = mg$ ....(i)

But from Universal Law of Gravitation,

$$F = \frac{GMm}{d^2} \quad ....(ii)$$

From equations (i) and (ii), we get:

$$mg = \frac{GMm}{d^2}$$

$$\Rightarrow \quad g = \frac{GM}{d^2}$$

Where M is the mass of the earth and d is the distance between the object and the earth.

For objects near or on the surface of the earth distance d is equal to the radius of the earth R.

Thus, $g = \frac{GM}{R^2}$ .....(iii)

Value of 'g' may vary at different parts of the earth –

- From the equation $g = GM/r^2$ it is clear that the value of 'g' depends upon the distance of the object from the earth's centre.
- This is because the shape of the earth is not a perfect sphere. It is rather flattened at poles and bulged out at the equator.
- Hence, the value of 'g' is greater at the poles and lesser at the equator. However, for our convenience, we take a constant value of 'g' throughout.

We can find the value of acceleration due to gravity by the following –
To Calculate the Value of g

Value of universal gravitational constant, $G = 6.7 \times 10^{-11}$ N m^2/ kg^2,
Mass of the earth, $M = 6 \times 10^{24}$ kg, and
Radius of the earth, $R = 6.4 \times 10^6$ m
Putting all these values in equation (iii), we get:

$$g = \frac{6.7\times10^{-11}\text{Nm}^2/\text{kg}^2\times6\times10^{24}\text{kg}}{\left(6.4\times10^6\text{m}\right)^2} = 9.8\,\text{m/s}^2$$

Thus, the value of acceleration due to gravity of the earth, $g = 9.8$ m/s^2

What is Free Fall?

When an object falls towards the earth due to earth's gravity and no other force is acting upon it, the object is said to be in **free fall state**. Free falling objects are not even resisted by the air.
$g = 9.8$ m/s^2 is also called the **Free-fall Acceleration**.

Value of 'g' is same on the earth, so the equations of motion for an object with uniform motion are valid where acceleration 'a' is replaced by 'g', as given under:

$v = u + gt$

$s = ut + (1/2)\, gt^2$

$2\,g\,s = v^2 - u^2$

Consider the equations of motion given in different scenarios:

When an object at rest falls towards earth – its initial velocity is zero v = gt s = t + (1/2) gt^2 2 g s = v^2
When an object with some initial velocity (u) falls towards earth – v = u + gt s = ut + (1/2) gt^2 2 g s = $v^2 - u^2$
When an object is thrown upwards from earth – the gravitational force acts in opposite direction, hence g is negative v = u - gt s = ut - (1/2) gt^2 -2 g s = $v^2 - u^2$

Difference between Universal gravitational Constant and Acceleration due to Gravity

S. No.	Gravitation Constant (G)	Gravitational acceleration (g)
1.	Its value is $6.67\times10^{-11}Nm^2/kg^2$.	Its value is 9.8 m/s^2.
2.	It is a scalar quantity.	It is a vactor quantity.
3.	Its value remains constant always and everywhere.	Its value varies at various places.
4.	Its unit is Nm^2/kg^2.	Its unit is m/s^2.

Mass & weight

Mass (m)

- The mass of a body is the quantity of matter contained in it.
- Mass is a scalar quantity which has only magnitude but no direction.
- Mass of a body always remains constant and does not change from place to place.
- SI unit of mass is kilogram (kg).
- Mass of a body can never be zero.

Weight (W)

- The force with which an object is attracted towards the centre of the earth, is called the weight of the object.
  Now, Force = $m \times a$
  But in case of earth, $a = g$
  $\therefore F = m \times g$
  But the force of attraction of earth on an object is called its weight (W).
  $\therefore W = mg$
- As weight always acts vertically downwards, therefore, weight has both magnitude and direction and thus it is a vector quantity.
- The weight of a body changes from place to place, depending on mass of object.
- The SI unit of weight is Newton.
- Weight of the object becomes zero if g is zero.

Weight of an Object on the Surface of Moon

Mass of an object is same on earth as well as on moon. But weight is different.

Distinguish between Mass and Weight

Mass	Weight
Mass of an object can be measured by its inertia.	Weight = mass × acceleration (m×g).
The total quantity of matter contained in an object is called mass of an object.	The gravitational force by which earth attracts an object is called weight of the object.
Mass of the object remains constant at all the places	Weight of the object is different at different places.
Measurement of mass is done by using a pan or beam balance.	Measurement of weight is done by using a spring balance.
Mass does not change even value of g is zero at any place.	Weight of the object becomes zero if g is zero.

Weight of an object on the Moon

Just like the Earth, the Moon also exerts a force upon objects. Hence, objects on moon also have some weight. The weight will not be same as than on the earth. So, weight on the Moon can be calculated as -

$$W_M = \frac{GM_M m}{R_M^2}$$

Now,

$$\Rightarrow \frac{W_M}{W_E} = \frac{M_M R_E^2}{M_E R_M^2}$$

Where,

$M_E = 5.98 \times 10^{24}\ kg$
$M_M = 7.36 \times 10^{22}\ kg$
$R_E = 6.4 \times 10^6\ m$
$R_M = 1.74 \times 10^6\ m$

$$\Rightarrow \frac{W_M}{W_E} = \frac{7.36 \times 10^{22} \times (6.4 \times 10^6)^2}{5.98 \times 10^{24} \times (1.74 \times 10^6)^2} = 0.165 \approx \frac{1}{6}$$

Therefore, weight of an object on the moon is $\frac{1}{6}$ of its weight on the Earth.